

THERMAL PROCESSES AT THE ANODE DURING AN INTENSE PULSED DISCHARGE

A. G. Goloveiko

UDC 537.50

The thermal processes during the initial and subsequent stages of the pulse are discussed on the basis of an account of surface evaporation and all the heat sources acting at the anode.

The operation of the anode in an electric discharge is simpler than that of the cathode, primarily because the anode does not emit any charged particles, but merely acts as an electron collector. The role of the anode is less important than that of the cathode in developing and maintaining the discharge.

Among studies of anode operation, those by Bez and Höcker occupy a special position [1-5]. Their theory of the elementary processes in the region of the anodic potential drop is in satisfactory agreement with experiment, and is generally accepted [6]. For an intense pulsed discharge, this theory leads to the following expression for the electronic heat flux density transferred to the anode:

$$F_e = (e\varphi' + eU_a + 2kT_e) \frac{j}{e} = (1 + \omega + \chi) \varphi' j. \quad (1)$$

For $U_a \approx U_1$, the ratio ω lies in the range 1.6-2.6 for various metals, while the coefficient χ may take on values in the range 0.4-1.9 for $T_e \approx (1-4) \cdot 10^4$ K. The quantity $(1 + \omega + \chi)\varphi'$ may vary within the range 5-10 V.

An intense pulsed discharge is accompanied by the passage of a discharge current of extremely high density through the electrodes and by the appearance of large temperature gradients there. Under such conditions, the thermal processes should be analyzed on the basis of a heat-conductivity equation taking into account Lenz-Joule and Thomson volume heat sources. Following Landau and Lifshits [7], we write this equation in the following form:

$$\frac{\partial(c_V T)}{\partial t} = \text{div}(\lambda \nabla T) + T \mathbf{j} \nabla \alpha + \rho j^2, \quad (2)$$

where we have chosen the sense of \mathbf{j} so that it is parallel to the electronic current.

The quantity α which appears in (2) has been interpreted on the basis of the quantum theory of metals; with certain simplifications, it may be written, as Ioffe has shown [8], in the following manner:

$$\alpha = \pi^2 k^2 T / e \epsilon_0. \quad (3)$$

Substituting (3) into (2), we find

$$\frac{\partial(c_V T)}{\partial t} = \text{div}(\lambda \nabla T) + \frac{\pi^2 k^2}{\epsilon_0 e} T \mathbf{j} \nabla T + \rho j^2. \quad (4)$$

Equation (4) is extremely complicated, and cannot be solved unless certain plausible assumptions are made to linearize it. If we focus on the process at the initial stage of the pulse, before the melting point is reached, then it is completely plausible to consider only average values of quantities c_V , λ , and ρ in this temperature range. Furthermore, using the Wiedemann-Franz law for the temperature ($T = \lambda \rho L_0^{-1}$), and using the notation

Belorussian Polytechnic Institute, Minsk. Translated from *Inzhenerno-Fizicheskii Zhurnal*, Vol. 15, No. 6, pp. 1000-1008, December, 1968. Original article submitted March 18, 1968.

© 1972 Consultants Bureau, a division of Plenum Publishing Corporation, 227 West 17th Street, New York, N. Y. 10011. All rights reserved. This article cannot be reproduced for any purpose whatsoever without permission of the publisher. A copy of this article is available from the publisher for \$15.00.

$$\frac{\lambda}{c_v} = a, \quad \frac{\rho}{c_v} = u, \quad \frac{\pi^2 k^2 \lambda}{\varepsilon_0 e L_0} = b, \quad (5)$$

we can convert Eq. (4) into the simpler form

$$\frac{\partial T}{\partial t} = a \Delta T + buj \nabla T + uj^2, \quad (6)$$

where Δ and ∇ are the Laplace and Hamiltonian differential operators.

In the one-dimensional approximation, the electronic current density and the temperature gradient are antiparallel for the anode, so Eq. (6) yields

$$\frac{\partial T(x', t)}{\partial t} = a \frac{\partial^2 T(x', t)}{\partial x'^2} - buj \frac{\partial T(x', t)}{\partial x'} + uj^2. \quad (7)$$

A uniform current-density distribution has been assumed here in the semiinfinite region ($0 \leq x' \leq \infty$) in order to increase the efficiency of the Lenz-Joule heat source in raising the temperature right at the anode surface. During the initial stage of the pulse, when there is essentially no evaporation, the following conditions hold at the anode:

$$-\lambda \frac{\partial T(0, t)}{\partial x'} = (1 + \omega + \chi) \varphi' j, \quad \frac{\partial T(\infty, t)}{\partial x'} = 0. \quad (8)$$

Initially, we have $T(x', 0) = T_0 = 285^\circ\text{K}$.

The problem (7), (8) can be written in the dimensionless form

$$\frac{\partial \theta(\xi, \tau)}{\partial \tau} = \frac{\partial^2 \theta(\xi, \tau)}{\partial \xi^2} - B \frac{\partial \theta(\xi, \tau)}{\partial \xi} + 1, \quad (9)$$

$$\frac{\partial \theta(0, \tau)}{\partial \xi} = -q, \quad \frac{\partial \theta(\infty, \tau)}{\partial \xi} = 0, \quad \theta(\xi, 0) = 0, \quad (10)$$

where

$$\begin{aligned} \xi &= A_\xi j x', \quad \tau = A_\tau j^2 t, \quad q = A_q (1 + \omega + \chi) \varphi', \\ \theta(\xi, \tau) &= \frac{T(x', t) - T_0}{T_{\text{mp}} - T_0}, \quad A_\xi = \sqrt{\frac{u}{a(T_{\text{mp}} - T_0)}}, \quad A_\tau = \frac{u}{T_{\text{mp}} - T_0}, \\ A_q &= \sqrt{\frac{a}{\lambda^2 u (T_{\text{mp}} - T_0)}}, \quad B = A_\xi b (T_{\text{mp}} - T_0). \end{aligned} \quad (11)$$

The metal characteristics appearing in these similarity criteria are shown in the accompanying Table 1.

Along with Eq. (9), we should discuss two other equations, with the same initial and boundary conditions (10):

$$\frac{\partial \theta(\xi, \tau)}{\partial \tau} = \frac{\partial^2 \theta(\xi, \tau)}{\partial \xi^2} + 1, \quad (12)$$

$$\frac{\partial \theta(\xi, \tau)}{\partial \tau} = \frac{\partial^2 \theta(\xi, \tau)}{\partial \xi^2}. \quad (13)$$

Here we have eliminated the Thomson heat source from the first equation and both the Thomson and Lenz-Joule sources from the second.

The problem (13), (10) determines the temperature field which arises at the anode as a result of the surface heat source alone. The temperature at the anode surface according to this problem is given by

$$\theta(0, \tau) = \frac{2q \sqrt{\tau}}{\sqrt{\pi}}, \quad (14)$$

from which we find the instant τ_0 at which the melting point is reached $\theta(0, \tau_0) = 1$:

$$\tau_0 = \frac{\pi}{4q^2}. \quad (15)$$

TABLE 1. Critical Constants of Certain Metals

Metal	$A\tau, \frac{m^4}{A^2 \cdot \text{sec}}$	$A\zeta, m/A$	Aq, V^{-1}	B
Cd	$2,58 \cdot 10^{-16}$	$3,38 \cdot 10^{-6}$	10,86	$9,98 \cdot 10^{-2}$
Zn	$9,80 \cdot 10^{-17}$	$1,61 \cdot 10^{-6}$	12,90	$2,76 \cdot 10^{-2}$
Al	$4,10 \cdot 10^{-17}$	$7,10 \cdot 10^{-7}$	10,00	$3,72 \cdot 10^{-3}$
Cu	$1,37 \cdot 10^{-17}$	$3,94 \cdot 10^{-7}$	6,96	$5,72 \cdot 10^{-3}$
W	$5,23 \cdot 10^{-18}$	$1,23 \cdot 10^{-8}$	1,75	$2,70 \cdot 10^{-1}$

The data shown in the accompanying Table 1 show that the quantity τ_0 is extremely small ($\tau_0 \ll 1$) for any possible values of the quantity $(1 + \omega + \chi)\varphi'$.

The problem (12), (10) determines the temperature field which arises at the anode as a result of the combined effects of the surface and Lenz-Joule volume heat sources, but without the Thomson source. The temperature at the anode surface, according to this problem, is given by

$$\theta(0, \tau) = \frac{2q \sqrt{\tau}}{\sqrt{\pi}} + \tau. \quad (16)$$

If, as before, we consider the process at time τ_0 , given by Eq. (15), then the difference between temperatures (16) and (14),

$$\Delta\theta_1(0, \tau_0) = \tau_0 \quad (17)$$

gives the additional temperature increase caused by the Lenz-Joule heat source at time τ_0 . When $\tau_0 \ll 1$, we find $\Delta\theta_1(0, \tau_0) \ll 1$; i.e., the additional temperature increase is much less than the melting point. This means that this heat source is inefficient during the initial stage of the process for any pulse conditions.

The problem (9), (10) determines the temperature field which arises at the anode during the combined operation of all three heat sources, including the Thomson source. The temperature at the anode surface according to this problem is given by

$$\theta(0, \tau) = \frac{2q \sqrt{\tau}}{\sqrt{\pi}} \sum_{n=0}^{\infty} \frac{(-1)^n \left(\frac{B}{2}\right)^{2n} \tau^n}{n!(2n+1)} + \frac{B^2 q \sqrt{\tau^3}}{\sqrt{\pi}} \sum_{n=0}^{\infty} \frac{(-1)^n \left(\frac{B}{2}\right)^{2n} \tau^n}{n!(2n+1)(2n+3)} + \frac{Bq\tau}{2} + \tau. \quad (18)$$

When $\tau \ll 1$, we may limit this discussion to the zeroth approximation:

$$\theta(0, \tau) = \frac{2q \sqrt{\tau}}{\sqrt{\pi}} + \frac{Bq\tau}{2} \left(1 + \frac{2B\sqrt{\tau}}{3\sqrt{\pi}}\right) + \tau. \quad (19)$$

The difference between temperatures (19) and (16) at time τ_0 ,

$$\Delta\theta_2(0, \tau_0) = \frac{Bq\tau_0}{2} \left(1 + \frac{2B\sqrt{\tau_0}}{3\sqrt{\pi}}\right) \quad (20)$$

determines the additional temperature increase due to the Thomson heat source. Since $\tau_0 \ll 1$, we see from the tabulated data that $\Delta\theta_2(0, \tau_0) \ll 1$; i.e., the increased temperature is again much less than the melting point. This means that the Thomson heat source is also inefficient under any pulse conditions during the initial stage of the process.

Accordingly, the predominant heat source is the surface source; this is true during the initial stage of the process regardless of the pulse conditions at the anode. However, there is no basis for extending this conclusion to the subsequent stages of the process, during which an intense surface evaporation will arise as the anode surface becomes hotter.

This evaporation causes an important change in the energetic balance at the boundary, causing the latter to move very rapidly; the power loss through evaporation and the velocity of the evaporation front are given by [9-11]

$$F_n = r_v v(t) = F_0 \exp \left[-\frac{T_m}{T(0, t)} \right],$$

$$v(t) = v_0 \exp \left[-\frac{T_m}{T(0, t)} \right], \quad (21)$$

where $T(0, t)$ is the temperature of the front whose coordinate is specified in a moving coordinate system tied to the front itself:

$$x = x' - \int_0^t v(t) dt. \quad (22)$$

The thermal problem for the anode during the subsequent stage of the pulse can be written in terms of this moving coordinate system as follows:

$$\frac{\partial T(x, t)}{\partial t} = a \frac{\partial^2 T(x, t)}{\partial x^2} + v(t) \frac{\partial T(x, t)}{\partial x} + uj^2 \exp(-\delta x), \quad (23)$$

$$-\lambda \frac{\partial T(0, t)}{\partial x} = (1 + \omega + \chi) \varphi' j - r_v v(t), \quad (24)$$

$$\frac{\partial T(\infty, t)}{\partial x} = 0, \quad T(x, 0) = T_0,$$

where j is the current density at the surface of the anode spot, and u is given by

$$u = u_{mp} [1 + \alpha_u [T(0, t) - T_{mp}]]. \quad (25)$$

This problem takes into account only the Lenz-Joule volume heat source. The spatial distribution of the intensity of this heat source in this one-dimensional problem is expressed in terms of the intensity at the evaporation front by means of the approximate distribution function $\exp(-\delta x)$, where the parameter δ is the reciprocal of the characteristic length for the process or of the depth of the microscopic holes $\delta \approx h^{-1}$, which can be determined experimentally. The relation established between the volume heat source and the characteristic dimensions of the process, and the comparatively reliable expression for the intensity of the source at the evaporation front, constitute the most important elements in this approximation; they play the governing role in the process. The approximation can be refined even further, but this is not necessary here.

Problem (23), (24) is nonlinear and has no analytic solution. A computer solution has been found for a uniform volume-source distribution ($\delta = 0$) [10, 11]. It turns out that at high current densities the high-temperature field excited by the surface heat source develops in an extremely thin surface layer, reaching a nearly steady state essentially instantaneously, especially at the evaporation front; the additional temperature increase due to the volume source lags behind considerably. Accordingly, the process may be treated in a somewhat different manner, under the assumption that the steady-state temperature and front velocity, $T(0, t) = T$ and $v(t) = v$, arise instantaneously at the start of the process, and then are stably maintained, while transient phenomena occur everywhere except at the evaporation front itself.

This formulation of the problem corresponds to

$$\frac{\partial T(x, t)}{\partial t} = a \frac{\partial^2 T(x, t)}{\partial x^2} + v \frac{\partial T(x, t)}{\partial x} + uj^2 \exp(-\delta x), \quad (26)$$

$$T(0, t) = T, \quad T(\infty, t) = T_0, \quad T(x, 0) = T_0, \quad (27)$$

which, after conversion to dimensionless form,

$$\frac{\partial \theta(\xi, \tau)}{\partial \tau} = \frac{\partial^2 \theta(\xi, \tau)}{\partial \xi^2} + \frac{\partial \theta(\xi, \tau)}{\partial \xi} + \mu \exp(-\beta \xi), \quad (28)$$

$$\theta(0, \tau) = 1, \quad \theta(\infty, \tau) = 0, \quad \theta(\xi, 0) = 0 \quad (29)$$

may be solved by an integral Laplace transformation:

$$\begin{aligned} \theta(\xi, \tau) = & \frac{1}{2} \exp(-\xi) \operatorname{erfc} \left(\frac{\xi}{2\sqrt{\tau}} - \frac{1}{2} \sqrt{\tau} \right) + \frac{1}{2} \operatorname{erfc} \left(\frac{\xi}{2\sqrt{\tau}} + \frac{1}{2} \sqrt{\tau} \right) - \frac{\mu}{2} \exp \left[- \left(\frac{1}{\sqrt{\gamma_2}} + \frac{1}{2} \right) \xi \right] \\ & \times \int_0^\tau \exp(-\gamma_1 \eta) \operatorname{erfc} \left(\frac{\xi}{2\sqrt{\eta}} - \frac{1}{2} \sqrt{\gamma_2 \eta} \right) d\eta - \frac{\mu}{2} \exp \left[\left(\frac{1}{\sqrt{\gamma_2}} - \frac{1}{2} \right) \xi \right] \\ & \times \int_0^\tau \exp(-\gamma_1 \eta) \operatorname{erfc} \left(\frac{\xi}{2\sqrt{\eta}} + \frac{1}{2} \sqrt{\gamma_2 \eta} \right) d\eta + \frac{\mu}{\gamma_1} [1 - \exp(-\gamma_1 \tau)] \exp(-\beta \xi), \end{aligned} \quad (30)$$

where

$$\begin{aligned} \tau &= \frac{v^2 t}{a}; \quad \xi = \frac{vx}{a}; \quad \theta(\xi, \tau) = \frac{T(x, t) - T_0}{T - T_0}; \\ \mu &= \frac{uaj^2}{v^2(T - T_0)}; \quad \beta = \frac{a\delta}{v}; \quad \gamma_1 = \beta(1 - \beta); \quad \gamma_2 = \frac{1}{4} - \gamma_1. \end{aligned} \quad (31)$$

This solution is interesting because it contains information about the duration of the transient process. Equation (30) shows that the temperature $\theta(\xi, \tau)$ begins to lose its time dependence at $\gamma_1\tau > 1$. From this condition, and according to (31), the transition time can be evaluated:

$$t_t > [\delta(v - a\delta)]^{-1}, \quad (32)$$

where $v > a\delta$.

When the process reaches a steady state, the problem (26), (27), with a boundary condition of the first kind, and problem (23), (24), with a boundary condition of the second kind, convert into the same steady-state problem:

$$a \frac{d^2 T(x)}{dx^2} + v \frac{dT(x)}{dx} + uj^2 \exp(-\delta x), \quad (33)$$

$$-\frac{dT(0)}{dx} = \frac{(1 + \omega + \chi)\varphi' j}{ac_v} - \frac{r_v v}{ac_v}, \quad \frac{dT(\infty)}{dx} = 0, \quad (34)$$

$$T(0) = T, \quad T(\infty) = T_0, \quad (35)$$

whose solution can be written

$$T(x) = \frac{u_{mp} [1 + \alpha_u (T - T_{mp})] j^2}{\delta(v - a\delta)} \left[\exp(-\delta x) - \exp\left(-\frac{vx}{a}\right) \right] + (T - T_0) \exp\left(-\frac{vx}{a}\right) + T_0, \quad (36)$$

$$v[r_v + c_v(T - T_0)] = (1 + \omega + \chi)\varphi' j + \frac{c_v u_{mp} [1 + \alpha_u (T - T_{mp})] j^2}{\delta}. \quad (37)$$

We introduce the following notation for the terms on the right side of this latter equation:

$$(1 + \omega + \chi)\varphi' j = F_e, \quad \frac{c_v u_{mp} [1 + \alpha_u (T - T_{mp})] j^2}{\delta} = F_w \quad (38)$$

and we consider the most important steady-state cases for the process.

1. We assume $F_w \ll F_e$ and $v \ll a\delta$; then it follows from (36) that

$$T(x) \approx (T - T_0) \exp\left(-\frac{vx}{a}\right) + T_0, \quad (39)$$

which shows that the volume heat source is completely ineffective when the steady-state regime is not limited in time. Specifying $v = 10^{-1}a\delta$, we find from $F_e + F_w \approx F_e$ and Eq. (37) the current density corresponding to these conditions:

$$j' = \frac{a\delta}{10(1 + \omega + \chi)\varphi'} \left[r_v + c_v \left(T_m \ln^{-1} \frac{10\alpha_u}{a\delta} - T_0 \right) \right]. \quad (40)$$

2. We assume $F_e = F_w$ or that the surface volume and heat sources are equivalent. Eliminating the temperature from Eq. (37) on the basis of $F_e = F_w$, we find the following transcendental equation for the corresponding current density j'' :

$$j''^2 \exp\left(\frac{j''}{A'' - B'' j''}\right) - C'' j'' = D'', \quad (41)$$

where

$$A'' = \frac{(1 + \omega + \chi)\varphi'\delta}{c_v u_{mp} \alpha_u T_m}; \quad B'' = \frac{1 - \alpha_u T_{mp}}{\alpha_u T_m}; \quad D'' = \frac{v_0 \delta}{2\alpha_u u_{mp}};$$

$$C'' = \frac{v_0}{2(1 + \omega + \chi)\varphi'} \left\{ r_v - \frac{c_v}{\alpha_u} [1 - \alpha_u (T_{mp} - T_0)] \right\}. \quad (42)$$

Since $F_e \sim j$ and $F_w \sim j^2$, the equality $F_e = F_w$ should be violated on both sides of j'' . When $j > j''$, we also have $F_w \gg F_e$ under the same condition, $v \gg a\delta$, so it follows from Eq. (36) that

$$T(x) \approx \frac{uj^2}{v\delta} \exp(-\delta x) + T_0, \quad (43)$$

for the electrode regions where $x > a/v$; this shows the predominance of the volume heat source and the complete independence of the temperature field from the thermal conductivity of the metal.

3. With j^* as the current density at which the temperature gradient at the evaporation front vanishes, and after the temperature is eliminated from Eqs. (34) and (37), the following transcendental equation is found for this current density:

$$(A^* + B^*j^*) \ln \frac{C^*}{j^*} + D^*j^* = E^*, \quad (44)$$

where

$$\begin{aligned} A^* &= (1 + \omega + \chi)\varphi' T_0 r_v^{-1}; & B^* &= u_{mp}(1 - \alpha_u T_{mp})\delta^{-1}, \\ C^* &= F_0[(1 + \omega + \chi)\varphi']^{-1}; & D^* &= u_{mp}\alpha_u T_m \delta^{-1}, \\ E^* &= (1 + \omega + \chi)\varphi' T_m r_v^{-1}. \end{aligned} \quad (45)$$

Physically, the current density j^* has the following meaning: when $j < j^*$, the heat flux still penetrates through the evaporation front and into the depth of the electrode; when $j = j^*$, this process stops; and when $j > j^*$, the heat flux moves in the opposite direction, from the electrode to the evaporation front. In this latter case, the temperature within the electrode exceeds that at the front; i.e., volume superheating occurs.

Calculations on the basis of handbook data [12-14] for the quantities appearing in Eqs. (40), (41), and (44) show that the current density j'' is greater than j' by 1.5-2 orders of magnitude and differs only very slightly from j^* . It is therefore sufficient to use the data only for the current density j^* ; this should be done for two regimes, with the surface heat source predominant in one and the volume heat source predominant in the second. Practical examples of such regimes are

- I. $(1 + \omega + \chi)\varphi' = 10V$; $\delta = 10^5 m^{-1}$; $h = 10 \mu$;
- II. $(1 + \omega + \chi)\varphi' = 5V$; $\delta = 5 \cdot 10^4 m^{-1}$; $h = 20 \mu$;

For such metals as Cu, Sn, and Pb, j^* values of $7.50 \cdot 10^7$, $3.20 \cdot 10^7$, and $2.50 \cdot 10^7$ A/cm² correspond to the first regime, and values of $1.66 \cdot 10^7$, $8.00 \cdot 10^6$, and $5.10 \cdot 10^6$ A/cm² correspond to the second. These data show that anode conditions (regime II) under which the volume heat source would be predominant at not too high a current density are completely plausible. Such an effect would be in principle impossible at any current density at the initial stage of the pulse but here the effect may occur and is a direct consequence of intense surface evaporation.

According to Eqs. (34) and (32), the steady-state front velocity and the transition time for $j = j^*$ can be evaluated from

$$v^* = (1 + \omega + \chi)\varphi' j^* r_v^{-1}, \quad (46)$$

$$t_t > r_v \{ \delta [(1 + \omega + \chi)\varphi' j^* - a\delta r_v] \}^{-1} = t_t^*. \quad (47)$$

For the same metals (Cu, Sn, and Pb), v^* values of 153, 164, and 234 m/sec correspond to the first regime, while values of 16.9, 20.5, and 24.0 m/sec correspond to the second. In other words, the velocity falls off by roughly an order of magnitude at the transition from the first to the second regime. The transition time, on the other hand, increases abruptly: for these metals the t^* values of $6.95 \cdot 10^{-8}$, $6.20 \cdot 10^{-8}$, and $3.45 \cdot 10^{-8}$ sec correspond to the first regime, while values of $1.62 \cdot 10^{-6}$, $1.01 \cdot 10^{-6}$, and $8.70 \cdot 10^{-7}$ sec correspond to the second. However, the transition in the second case is completed quite rapidly in comparison with the total duration of pulses achievable in practice.

The steady-state role of the Thomson heat source can be determined from the heat-conductivity equation

$$a \frac{d^2 T(x)}{dx^2} + \left[v - buj \exp\left(-\frac{\delta x}{2}\right) \right] \frac{dT(x)}{dx} + uj^2 \exp(-\delta x) = 0, \quad (48)$$

which can be found from (7) on the basis of (22) under the assumption that the current density distribution corresponds to the approximate distribution adopted for the intensity of the Lenz–Joule heat source. In this equation, the inequality

$$v > buj, \quad (49)$$

holds, so Eq. (48) may be replaced by Eq. (33), and the Thomson heat source is seen to be ineffective in the steady state.

NOTATION

e	is the electronic charge;
φ'	is the work function;
U_i	is the ionization potential for the anode atoms;
U_a	is the anodic potential drop;
k	is the Boltzmann constant;
T_e	is the electronic plasma temperature in the region of the anodic potential drop;
j	is the anodic current density at the anode surface;
ε_0	is the electron energy at the Fermi level;
λ	is the thermal conductivity of the electrode;
c_V	is the specific heat capacity of the electrode material at constant volume;
ρ	is the resistivity of the electrode;
L_0	is the Lorentz number;
r_V	is the specific volume heat of vaporization of the electrode material;
v_0	is the velocity of sound in the electrode material;
u_{mp}	is the value of u (5) for the liquid phase of the electrode at the melting point;
α_u	is the temperature coefficient for a linear change of u in the liquid state of the metal;
T_{mp}	is the melting point of the electrode.

LITERATURE CITED

1. W. Bez and K. H. Höcker, *Z. f. Naturforsch.*, **9a**, 72 (1954).
2. K. H. Höcker and W. Bez, *Z. f. Naturforsch.*, **10a**, 706 (1955).
3. W. Bez and K. H. Höcker, *Z. f. Naturforsch.*, **10a**, 714 (1955).
4. W. Bez and K. H. Höcker, *Z. f. Naturforsch.*, **11a**, 118 (1956).
5. W. Bez and K. H. Höcker, *Z. f. Naturforsch.*, **11a**, 192 (1956).
6. W. Finkelnburg and H. Mekker, *Electric Arcs and Thermal Plasma* [Russian translation], IL (1961).
7. L. D. Landau and E. M. Lifshits, *Electrodynamics of Continuous Media* [in Russian], Moscow (1959).
8. A. F. Ioffe, *Semiconducting Thermocouples* [in Russian], Izd. Akad. Nauk SSSR, Moscow–Leningrad (1960).
9. S. I. Anisimov, A. M. Bonch-Bruevich, M. A. El'yashevich, Ya. A. Imas, N. A. Pavlenko, and G. S. Romanov, *Zh. Teor. Fiz.*, **36**, 1273 (1966).
10. A. G. Goloveiko, *Izvestiya Vuz., Énergetika*, No. 6, 83 (1966).
11. A. G. Goloveiko, *Izvestiya Vuz., Énergetika*, No. 5, 79 (1968).
12. A. N. Nesmeyanov, *Vapor Pressures of the Chemical Elements* [in Russian], Izd. Akad. Nauk SSSR (1961).
13. M. P. Slavinskii, *The Physicochemical Properties of the Elements* [in Russian], Metallurgizdat, Moscow (1952).
14. A. I. Gubanov, *Quantum-Electronic Theory of Amorphous Semiconductors* [in Russian], Izd. Akad. Nauk SSSR, Moscow–Leningrad (1963).